Math 70 12.4 — More Properties of Logarithms

Lessons 38-39 (1.5 days)

Objectives

combine-type problems

1) Combine $\log_b a + \log_b c$ using property
2) Combine $\log_b a - \log_b c$ using property
3) Repeatedly combine $\log_b a + \log_b a + \dots = k \log_b a$
4) Use these properties together to combine to a single log with coefficient 1.

separate-type problems.

5) Write a single log as a sum, difference and/or multiple of simpler logs using the same properties, in the opposite direction.
6) Given numerical values for a few simple logs, separate a more complicated log into those simpler logs to find its numerical value.

Math 70 (More) Properties of Logarithms, Day 1

Consider the sum of two logs, same base, b:

$$\log_b x + \log_b y$$

Let's make a substitution to make this simpler.

$M = \log_b x \longleftrightarrow b^M = x$

$N = \log_b y \longleftrightarrow b^N = y$

$$= M + N$$

Using the inverse property:

$$= \log_b b^{M+N}$$

Rules of exponents tell us:

$$= \log_b (b^M \cdot b^N)$$

Substitute back:

$$= \log_b (x \cdot y)$$

$$\boxed{\begin{array}{l}\text{Beginning} = \text{End:}\\ \log_b x + \log_b y = \log_b xy\end{array}}$$

← Sometimes called the Product property of logs.

Similarly:

$$\log_b x - \log_b y$$

$$= M - N$$

$$= \log_b b^{M-N}$$

$$= \log_b \left(\frac{b^M}{b^N}\right)$$

$$= \log_b \left(\frac{x}{y}\right).$$

Giving

$$\boxed{\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)}$$

← Sometimes called the Quotient Property of Logs.

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Remember $x + x + x = 3x$ multiplication is repeated addition.

So $\log_b x + \log_b x + \log_b x = 3\log_b x$.

Using the same methods as before:

$$3\log_b x$$

$= 3M$ Subst

$= \log_b b^{3M}$ Inverse property.

$= \log_b((b^M)^3)$ exponent rules.

$\boxed{\log_b x = M \text{ means } b^M = x.}$

$= \log_b (x^3)$. subst back

This means

$$3\log_b x = \log_b x^3$$

We can expand this for any multiple k:

$\boxed{k \cdot \log_b x = \log_b x^k}$ This is sometimes called the Power Rule for exponents.

Write each sum as a single log.

① $\log_{11} 10 + \log_{11} 3 = \log_{11}(10\cdot3) = \boxed{\log_{11} 33}$

② $\log_3 \frac{1}{2} + \log_3 12 = \log_3\left(\frac{1}{2}\cdot 12\right) = \boxed{\log_3 6}$

③ $\log_2(x+2) + \log_2 x = \log_2[x\cdot(x+2)] = \boxed{\log_2(x^2+2x)}$

Write each difference as a single log.

(4) $\log_{10} 27 - \log_{10} 3 = \log_{10}\left(\frac{27}{3}\right) = \boxed{\log_{10}(9)}$

(5) $\log_5 8 - \log_5 x = \boxed{\log_5 \frac{8}{x}}$

(6) $\log_3 (x^2+5) - \log_3(x^2+1) = \boxed{\log_3 \frac{x^2+5}{x^2+1}}$

Write each multiple as a single log.

(7) $2\log_7 3 = \log_7 3^2 = \boxed{\log_7 9}$

(8) $4\log_3 x = \boxed{\log_3 x^4}$

(9) $-5\log_9 2 = \log_9 2^{-5} = \boxed{\log_9 \frac{1}{32}}$

In MathXL, do all 9. problems asking you to write as a single log.

(10) $2\log_5 3 + 3\log_5 2 = \log_5 3^2 + \log_5 2^3 = \log_5 9\cdot 8 = \boxed{\log_5 72}$

(11) $3\log_9 x - \log_9(x+1) = \log_9 x^3 - \log_9(x+1) = \boxed{\log_9 \frac{x^3}{x+1}}$

(12) $\log_4 25 + \log_4 3 - \log_4 5 = \log_4\left(\frac{25\cdot 3}{5}\right) = \boxed{\log_4 15}$

Extras

(13) $5\log_6 x - \frac{3}{4}\log_6 x + 3\log_6 x = \log_6 x^5 - \log_6 x^{3/4} + \log_6 x^3 = \log_6 \frac{x^5\cdot x^3}{x^{3/4}}$

$= \log_6 \frac{x^8}{x^{3/4}}$

$= \boxed{\log_6 x^{29/4}}$

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(14) $2\log_5 x + \frac{1}{3}\log_5 x - 3\log_5(x+5)$

$= \log_5 x^2 + \log_5 x^{1/3} - \log_5 (x+5)^3$

$= \log_5 \frac{x^2 \cdot x^{1/3}}{(x+5)^3}$

add exponents

$2 + \frac{1}{3}$

$= \frac{6}{3} + \frac{1}{3}$

$= \frac{7}{3}$

$= \boxed{\log_5 \frac{x^{7/3}}{(x+5)^3}}$

(15) $2\log_7 y + 6\log_7 z$

$= \log_7 y^2 + \log_7 z^6$

$= \boxed{\log_7 (y^2 z^6)}$

SUMMARY OF LOG PROPERTIES $b \neq 1,\ b > 0$

1) $\log_b 1 = 0$

2) $\log_b b^x = x$

3) $b^{\log_b x} = x$

4) $\log_b x \cdot y = \log_b x + \log_b y$

5) $\log_b \frac{x}{y} = \log_b x - \log_b y$

6) $\log_b x^k = k \cdot \log_b x$

Math 70 "Separate"-type problems

Write each log as a sum of logs.

① $\log_3(20)$

<u>step 1</u>: Factor 20 to prime factors

$$\begin{array}{ccccc} & & & 20 & \\ & & 4 & & 5 \\ & 2 & & 2 & \end{array}$$

$$20 = 2^2 \cdot 5$$

$$\log_3(20) = \log_3(2^2 \cdot 5)$$

<u>step 2</u>: use properties 1 & 2 first

$$= \log_3(2^2) + \log_3(5)$$

<u>step 3</u>: use property 3 last

$$= \boxed{2\log_3(2) + \log_3(5)}$$

or $\boxed{2\log_3 2 + \log_3 5}$

② $\log_2(4y) = \log_2(2^2 \cdot y)$

$$= \log_2(2^2) + \log_2 y$$

← remember inverse functions!

$$\log_2(2^x) = x$$

$$= \boxed{2 + \log_2 y}$$

③ $\log_5(ab) = \boxed{\log_5 a + \log_5 b}$

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Write each log as a difference of logs

(4) $\log_3\left(\frac{5}{4}\right) = \log_3(5) - \log_3(4)$

$= \log_3(5) - \log_3(2^2)$

$= \boxed{\log_3 5 - 2\log_3 2}$

(5) $\log_2\left(\frac{8}{y}\right) = \log_2(8) - \log_2(y)$

$= \log_2(2^3) - \log_2(y)$ inverses again!

$= \boxed{3 - \log_2 y}$ $\log_2 2^x = x$

(6) $\log_5\left(\frac{a}{b}\right) = \boxed{\log_5 a - \log_5 b}$

Write each log as the multiple of a log

(7) $\log_3 x^5 = \boxed{5 \cdot \log_3 x}$ or $\boxed{5\log_3 x}$

(8) $\log_4 \sqrt{x} = \log_4 x^{1/2}$ remember $\sqrt{x} = x^{1/2}$

$= \boxed{\frac{1}{2}\log_4 x}$

(9) $\log_7(a^b) = \boxed{b \cdot \log_7 a}$

Write each log as the sum and/or difference of multiples of logs.

(10) $\log_3\left(\frac{35}{44}\right) = \log_3\left(\frac{5 \cdot 7}{2^2 \cdot 11}\right)$ find prime factors

$= \log_3 5 + \log_3 7 - \log_3 2^2 - \log_3 11$

$= \boxed{\log_3 5 + \log_3 7 - 2\log_3 2 - \log_3 11}$

factors in numerator are added; factors in denom are subtracted

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(11) $\log_2 \frac{x^5}{(y+1)^2} = \log_2 x^5 - \log_2 (y+1)^2$

$= \boxed{5\log_2 x - 2\log_2(y+1)}$

Note: parentheses are required.
$\log_2 y+1$ means $\log_2(y) + 1$
or $\log_2(y) + \log_2(2)$

(12) $\log_3\left(\frac{x+5}{x}\right)^2$

option 1: outside exponent first

$= 2\log_3\left(\frac{x+5}{x}\right)$

$= 2\left[\log_3(x+5) - \log_3(x)\right]$

$= \boxed{2\log_3(x+5) - 2\log_3(x)}$

option 2: use exponent properties first

$= \log_3 \frac{(x+5)^2}{x^2}$

$= \log_3(x+5)^2 - \log_3 x^2$

$= \boxed{2\log_3(x+5) - 2\log_3(x)}$

parentheses required

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Given $\log_b 2 = 0.43$
and $\log_b 3 = 0.68$,
Use properties of logs to evaluate the following:

← Notice: base b not given. Don't know any logs except $\log_b 2$ and $\log_b 3$.

(13) $\log_b 6$

step 1: Write argument in prime factors and powers

$6 = 2 \cdot 3$

step 2: Rewrite log using prime factors

$= \log_b 2 \cdot 3$

step 3: Write as sum/difference of multiples of logs.

$= \log_b 2 + \log_b 3$

step 4: Substitute the given values

$$= \underbrace{\log_b 2}_{\downarrow} + \underbrace{\log_b 3}_{\downarrow}$$

$= 0.43 \qquad 0.68$

step 5: Do arithmetic (must be easy – if you get a mile-long decimal, you made a mistake!)

$= \boxed{1.11}$

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(14) $\log_b 9 = \log_b 3\cdot 3$ or $\log_b 3^2$

$= \log_b 3 + \log_b 3$

$= .68 + .68$

$= \boxed{1.36}$

$= 2\log_b 3$

$= 2(0.68)$

$= \boxed{1.36}$

(15) $\log_b \sqrt{2}$

$= \log_b 2^{1/2}$

$= \frac{1}{2}\log_b 2$

$= \frac{1}{2}(.43)$

$= \boxed{0.215}$

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same instructions

(16) $\log_b 36$

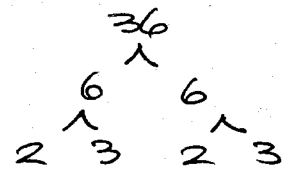
$= \log_b(2^2 \cdot 3^2)$

$= \log_b 2^2 + \log_b 3^2$

$= 2\log_b 2 + 2\log_b 3$

$= 2(0.43) + 2(0.68)$

$= \boxed{2.22}$



$36 = 2^2 \cdot 3^2$

(17) $\log_b 72$

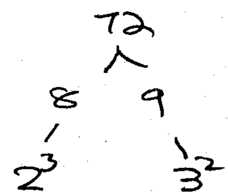
$= \log_b(2^3 \cdot 3^2)$

$= \log_b 2^3 + \log_b 3^2$

$= 3\log_b 2 + 2\log_b 3$

$= 3(0.43) + 2(0.68)$

$= \boxed{2.65}$



(18) $\log_b \frac{4}{9}$

$= \log_b \frac{2^2}{3^2}$

$= \log_b 2^2 - \log_b 3^2$

$= 2\log_b 2 - 2\log_b 3$

$= 2(.43) - 2(.68)$

$= \boxed{-0.5}$

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same instructions, continued.

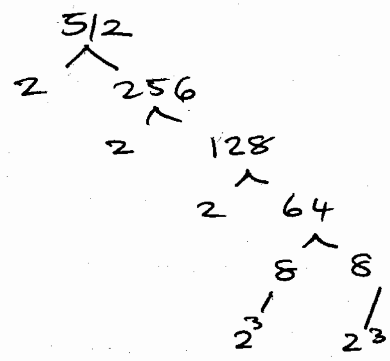
(19) $\log_b 512$

$= \log_b 2^9$

$= 9 \log_b 2$

$= 9(.43)$

$= \boxed{3.87}$



Challenge Problem

(20) $\log_b \sqrt[3]{1536}$

$= \log_b (1536)^{1/3}$

$= \frac{1}{3} \log_b 1536$

$= \frac{1}{3} \log_b (2^9 \cdot 3)$

$= \frac{1}{3} \log_b 2^9 + \frac{1}{3} \log_b 3$

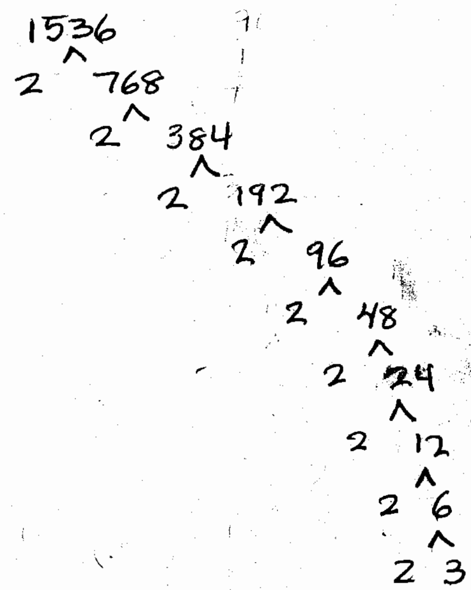
$= 9 \cdot \frac{1}{3} \cdot \log_b 2 + \frac{1}{3} \log_b 3$

$= 3 \log_b 2 + \frac{1}{3} \log_b 3$

$= 3(.43) + \frac{1}{3}(.68)$

$= \boxed{1.51\overline{6}}$ ← Do not round!

$= \boxed{\frac{91}{60}}$



Math 70 MG 5/e 9.6 Day 2

Review:

Write as a single log

(20) $\log_6 18 + 3\log_6 2 - \log_6 9$

$= \log_6 18 + \log_6 2^3 - \log_6 9$

$= \log_6 18 + \log_6 8 - \log_6 9$

$= \log_6 \left(\frac{18 \cdot 8}{9}\right)$

$= \boxed{\log_6 16}$

(21) $\log_9 4x - \log_9 (x-3) + \log_9 (x^3+1)$

$= \boxed{\log_9 \frac{4x(x^3+1)}{x-3}}$

Write as sum or difference or multiple logs

(22) $\log_6 \frac{x^2}{x+3}$

$= \log x^2 - \log_6 (x+3)$

$= \boxed{2\log x - \log_6 (x+3)}$

(24) $\log_2 \frac{x^3}{\sqrt{y}}$

$= \log_2 x^3 - \log_2 \sqrt{y}$

$= \boxed{3\log_2 x - \frac{1}{2}\log_2 y}$

(23) $\log_5 x^3(x+1)$

$= \log_5 x^3 + \log_5 (x+1)$

$= \boxed{3\log_5 x + \log_5 (x+1)}$